6.1100

Register Allocation

Outline

- What is register allocation
- Webs
- Interference Graphs
- Graph coloring
- Spilling
- Splitting
- More optimizations

Storing values between def and use

- Program computes with values
  - value definitions (where computed)
  - value uses (where read to compute new values)
- Values must be stored between def and use
  - First Option
    - store each value in memory at definition
    - retrieve from memory at each use
  - Second Option
    - store each value in register at definition
    - retrieve value from register at each use

Register Allocation

- Deciding which values to store in limited number of registers
- Register allocation has a direct impact on performance
  - Affects almost every statement of the program
  - Eliminates expensive memory instructions
  - # of instructions goes down due to direct manipulation of registers
    - Limited mem-to-mem ALU ops, may need two instructions
  - Probably is the optimization with the most impact!

What can be put in a register?

- Values stored in compiler-generated temps
- Language-level values
  - Values stored in local scalar variables
  - Big constants
  - Values stored in array elements and object fields
    - Issue: alias analysis
- Register set depends on the data-type
  - floating-point values in floating point registers
  - integer and pointer values in integer registers

Issues

- Fewer instructions when using registers
  - Additional instructions when using memory accesses
- Registers are faster than memory
  - wider gap in faster, newer processors
  - Factor of about 4 bandwidth, factor of about 3 latency
  - Could be bigger if program characteristics were different
- But only a small number of registers available
  - Usually 16 integer and 16 floating-point registers
  - Some of those registers have fixed users (ex: RSP, RBP)

Outline

- What is register allocation
- Key ideas in register allocation
- Webs
- Interference Graphs
- Graph coloring
- Splitting
- More optimizations

Summary of Register Allocation

- You want to put each temporary in a register
  - *But*, you don't have enough registers.
- Key Ideas:
  - When a temporary goes dead, its register can be reused
  - Two live temporaries can't use the same register at the same time

Summary of Register Allocation

- When a temporary goes dead, its register can be reused
- Example:

```
a := c + d
e := a + b
f := e - 1
```

(assume that a and e die after use)

- temporaries a, e and f can go in the same register

```
r1 := c + d
r1 := r1 + b
r1:= r1 – 1
```

Summary of Register Allocation

- Two live temporaries can't use the same register at the same time
- Example 2:

```
a := c + d
e := a + b
f := e - a
```

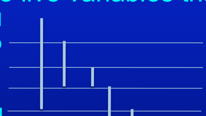
- temporaries e and a can not go in the same register

```
r1 := c + d
r2 := r1 + b
r1 := r2 – r1
```

When things don't work out

- Sometimes more live variables than registers

```
a := c + d
e := c + b
f := e – c
g := e + f
h := a + g
```



Won't work for 2 registers

(assume only g and h live at the end)

- You can split a live range by storing to memory

```
a := c + d
store a
e := c + b
f := e – c
g := e + f
load a
h := a + g
```

Web-Based Register Allocation

- Determine live ranges for each value (*web*)
- Determine overlapping ranges (interference)
- Compute the benefit of keeping each web in a register (spill cost)
- Decide which webs get a register (allocation)
- Split webs if needed (spilling and splitting)
- Assign hard registers to webs (assignment)
- Generate code including spills (code gen)

Outline

- What is register allocation
- Key ideas in register allocation
- Webs
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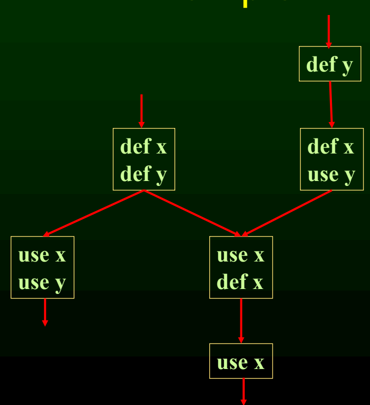
Webs

- Starting Point: def-use chains (DU chains)
  - Connects definition to all reachable uses
- Conditions for putting defs and uses into same web
  - Def and all reachable uses must be in same web
  - All defs that reach same use must be in same web
- Use a union-find algorithm

Example



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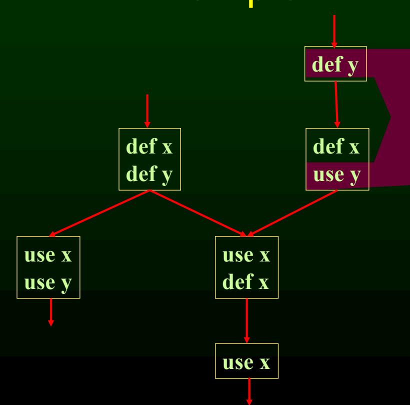

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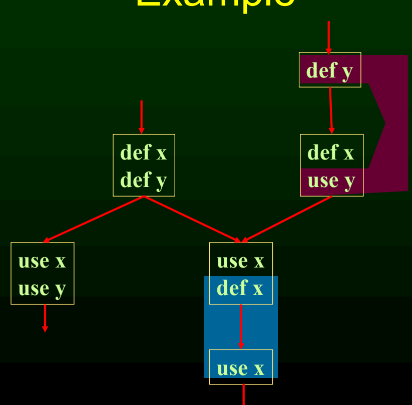

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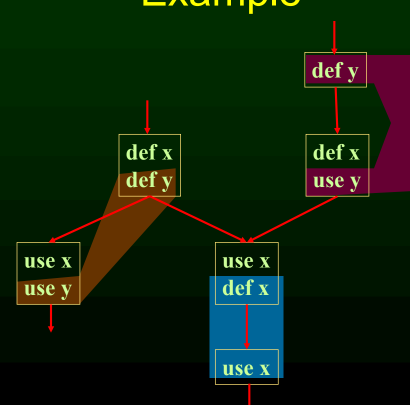

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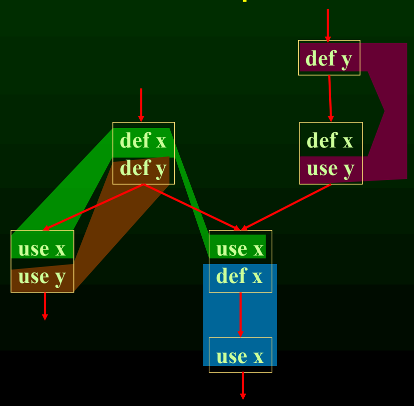

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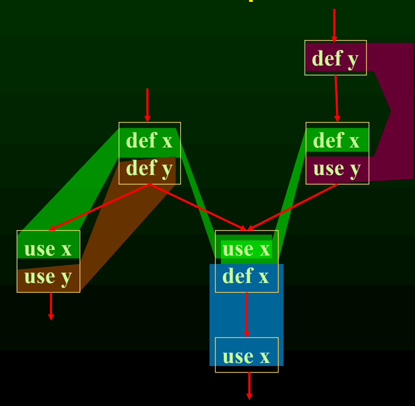

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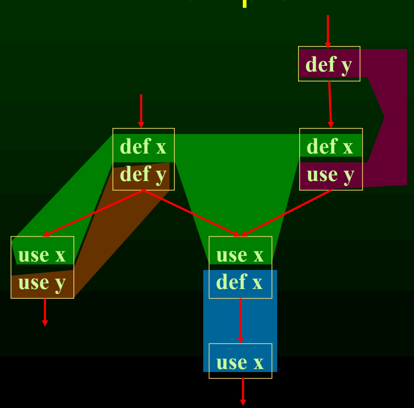

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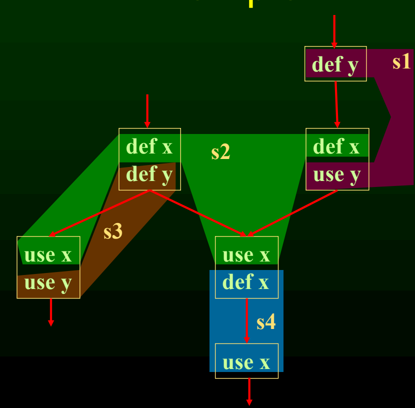


Webs

- Web is unit of register allocation
- If web allocated to a given register R
  – All definitions computed into R
  – All uses read from R
- If web allocated to a memory location M
  – All definitions computed into M
  – All uses read from M

Outline

- What is register allocation
- Webs
- Interference Graphs
- Graph coloring
- Splitting
- More optimizations

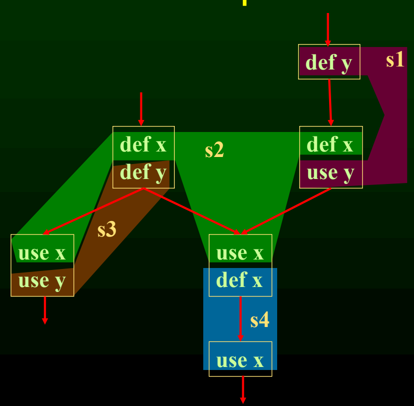
Convex Sets and Live Ranges

- Concept of convex set
- A set S is convex if
  - A, B in S and C is on a path from A to B implies
  - C is in S
- Concept of live range of a web
  - Minimal convex set of instructions that includes all defs and uses in web
  - Intuitively, region in which web's value is live

Interference

- Two webs interfere if their live ranges overlap (have a nonemtpy intersection)
- If two webs interfere, values must be stored in different registers or memory locations
- If two webs do not interfere, can store values in same register or memory location

Example



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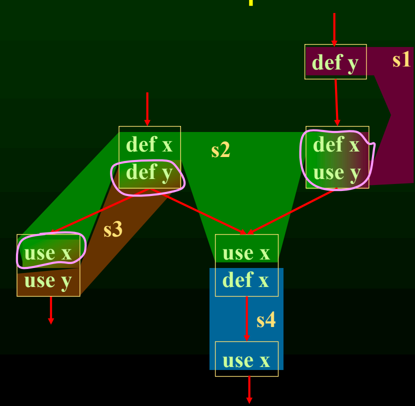
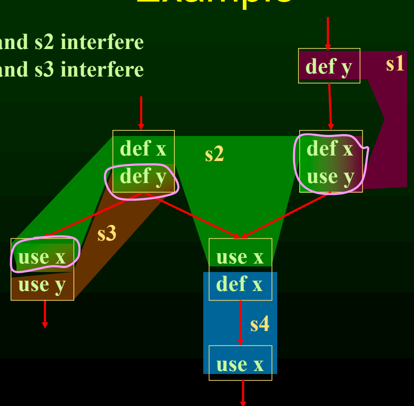


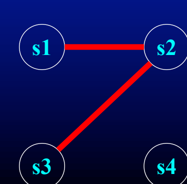
Example

Webs s1 and s2 interfere
Webs s2 and s3 interfere



Interference Graph

- Representation of webs and their interference
  - Nodes are the webs
  - An edge exists between two nodes if they interfere



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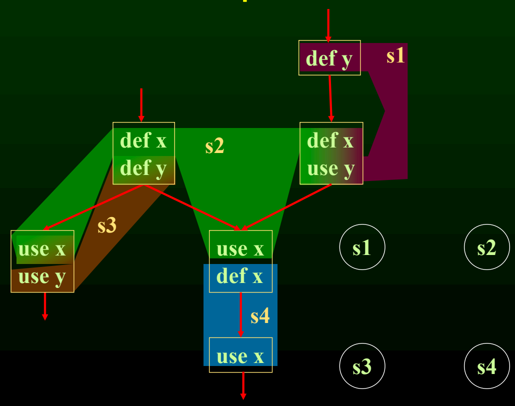
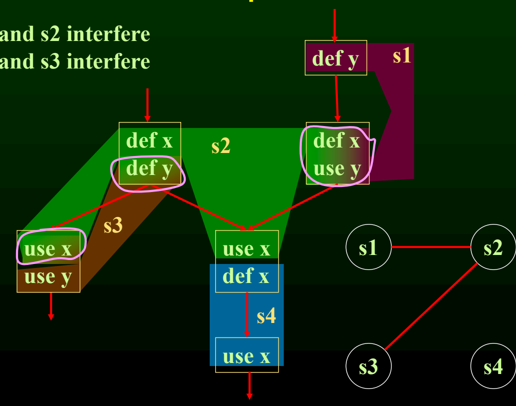


Example

Webs s1 and s2 interfere
Webs s2 and s3 interfere

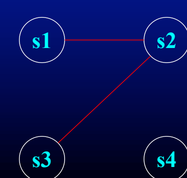


Outline

- Overview of procedure optimizations
- What is register allocation
- A simple register allocator
- Webs
- Interference Graphs
- Graph coloring
- Splitting
- More optimizations

Register Allocation Using Graph Coloring

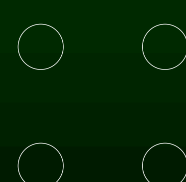
- Each web is allocated a register
  - each node gets a register (color)
- If two webs interfere they cannot use the same register
  - if two nodes have an edge between them, they cannot have the same color



Graph Coloring

- Assign a color to each node in graph
- Two nodes connected to same edge must have different colors
- Classic problem in graph theory
- NP complete
  - But good heuristics exist for register allocation

Graph Coloring Example



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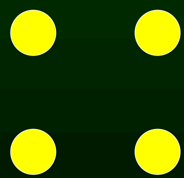
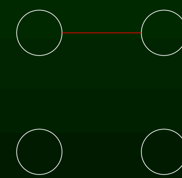
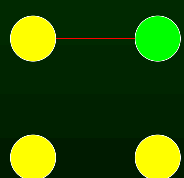


- **1 Color**

Graph Coloring Example

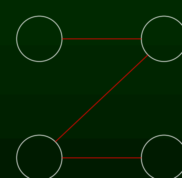


Graph Coloring Example

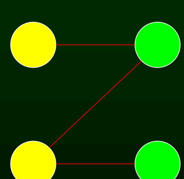


- **2 Colors**

Graph Coloring Example

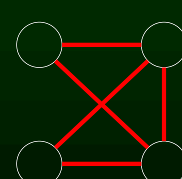


Graph Coloring Example



- **Still 2 Colors**

Graph Coloring Example



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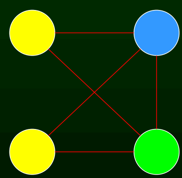


- 3 Colors

Heuristics for Register Coloring

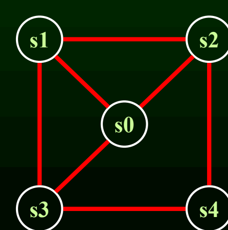
- Coloring a graph with N colors
- If degree < N (degree of a node = # of edges)
  - Node can always be colored
  - After coloring the rest of the nodes, you'll have at least one color left to color the current node
- If degree >= N
  - still may be colorable with N colors

Heuristics for Register Coloring

- Remove nodes that have degree < N
  - push the removed nodes onto a stack
- When all the nodes have degree >= N
  - Find a node to spill (no color for that node)
  - Remove that node
- When empty, start to color
  - pop a node from stack back
  - Assign it a color that is different from its connected nodes (since degree < N, a color should exist)

Coloring Example

N = 3



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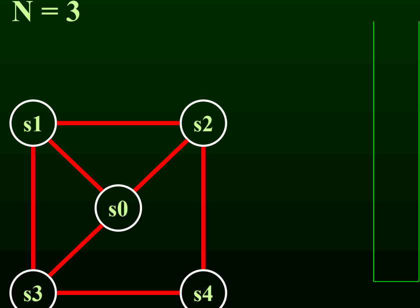

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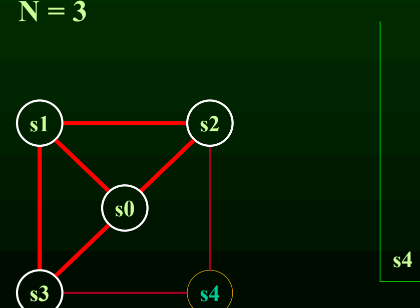

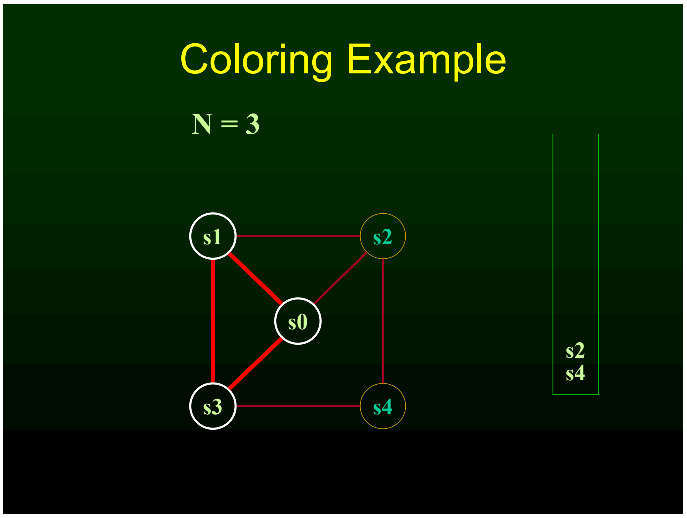
Coloring Example
N = 3
s1
s2
s0
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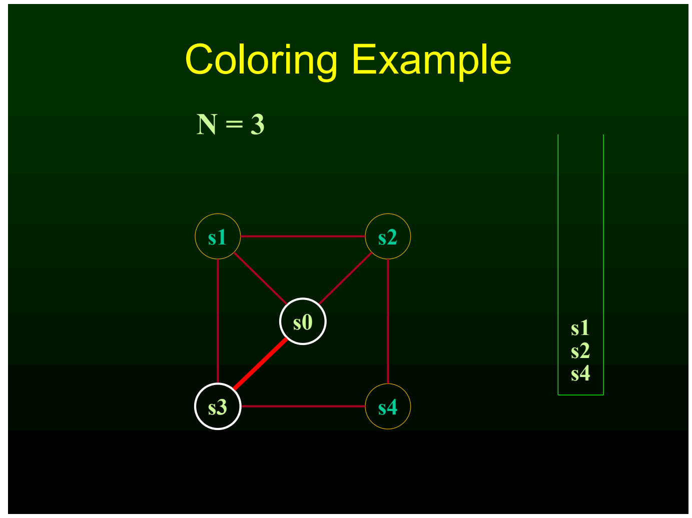
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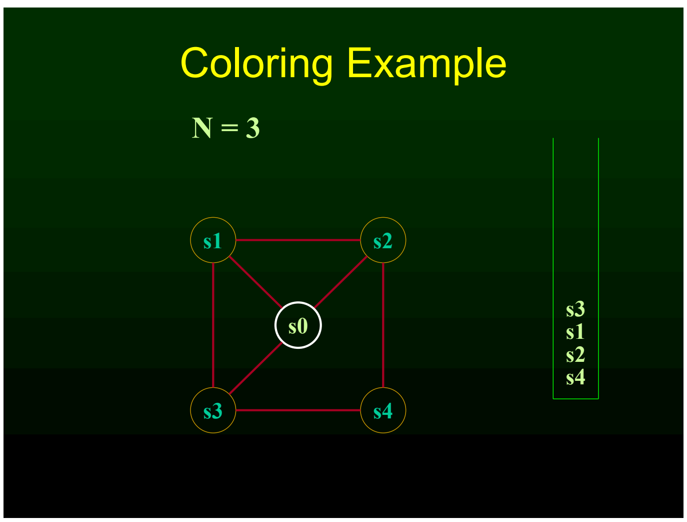
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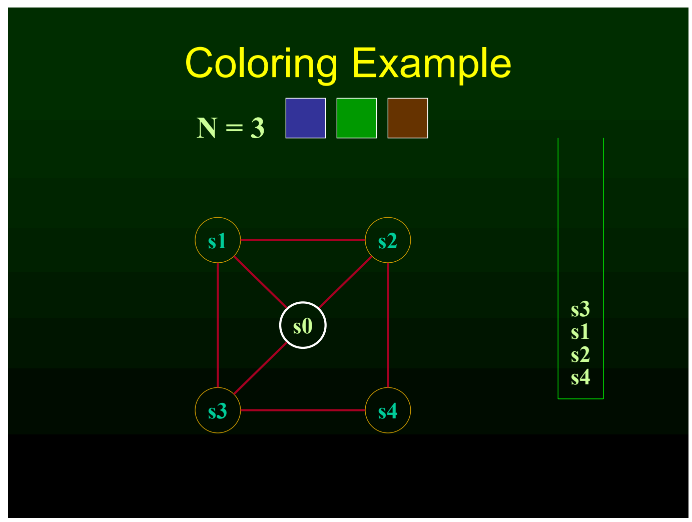
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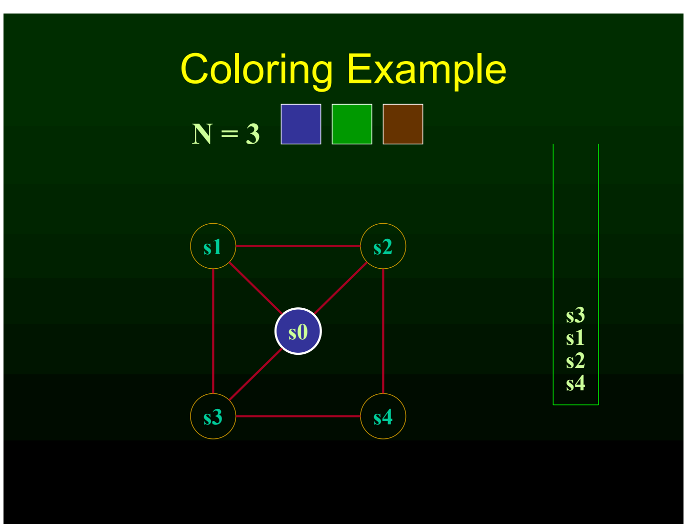
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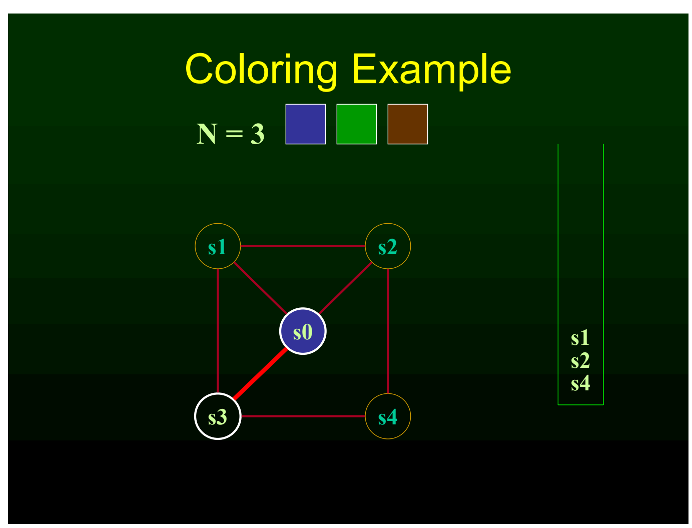
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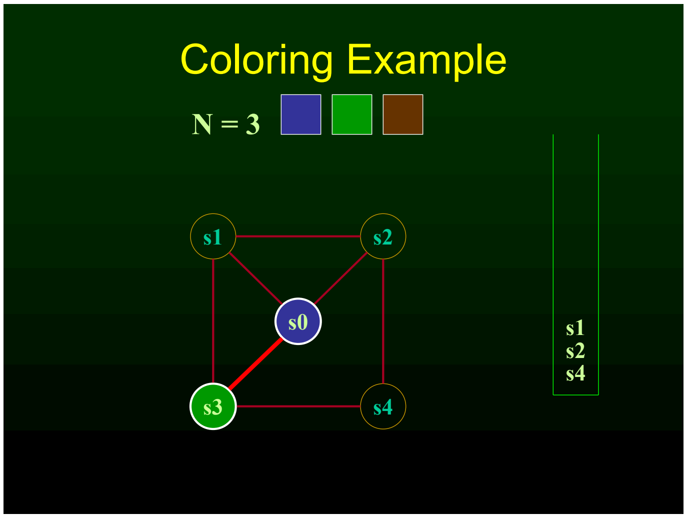
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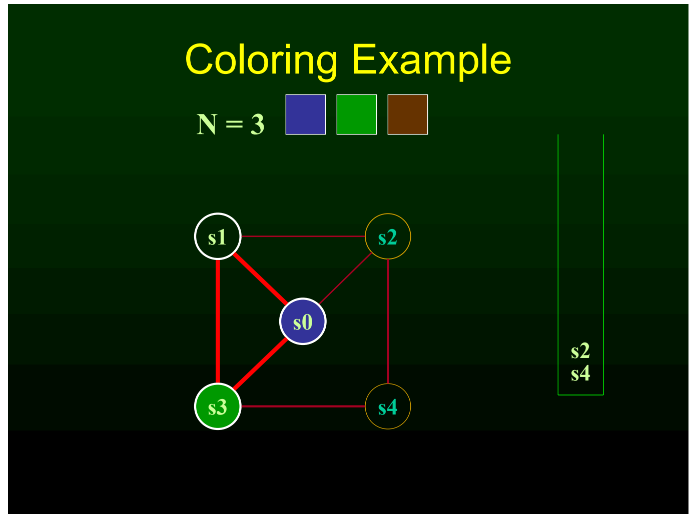
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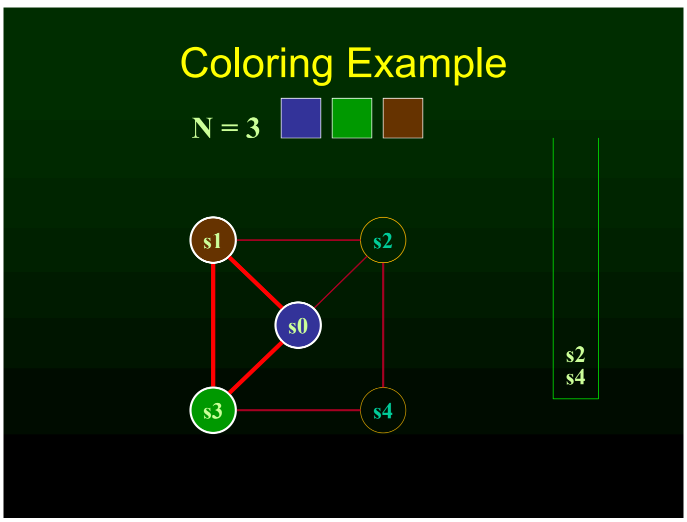
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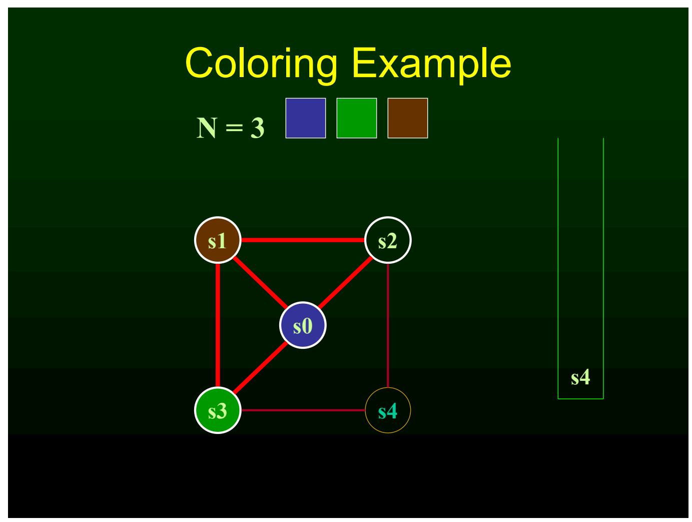
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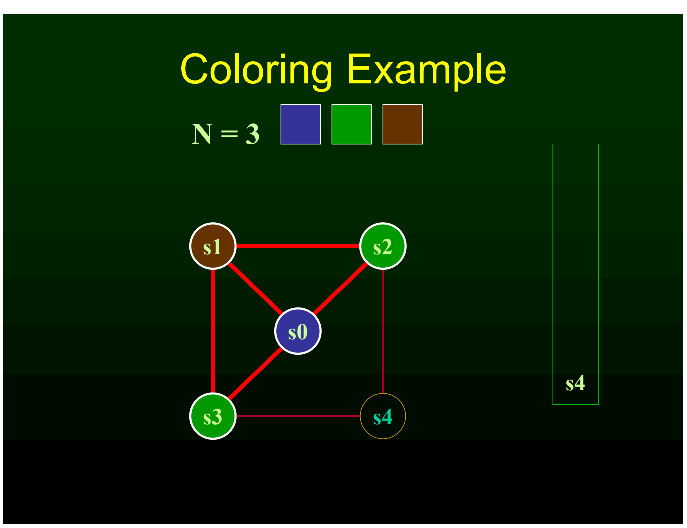
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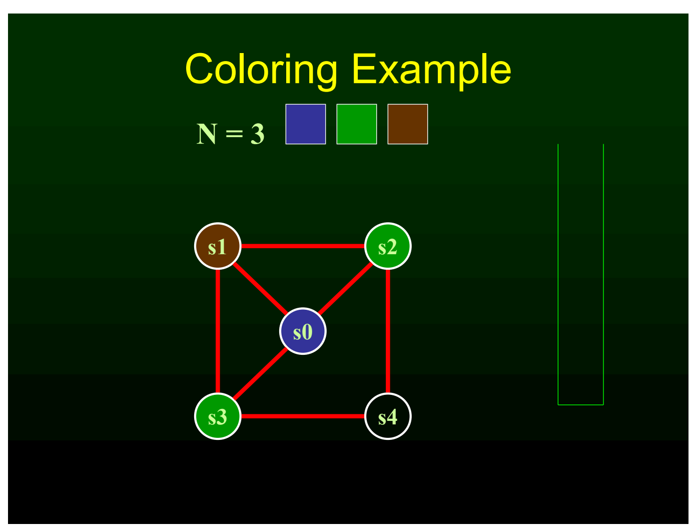
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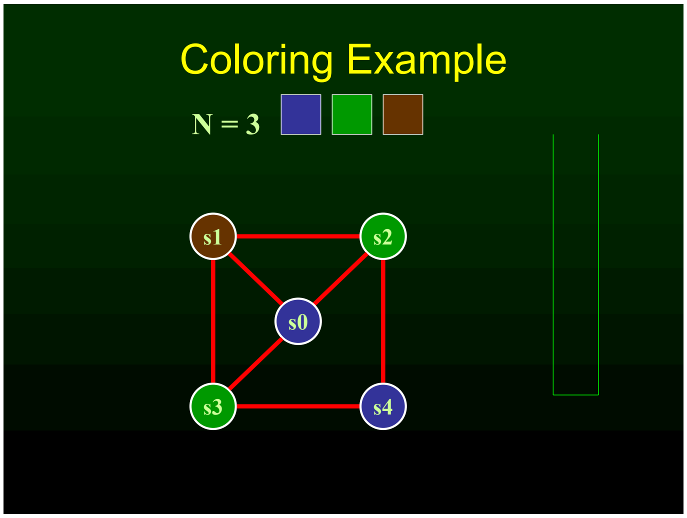
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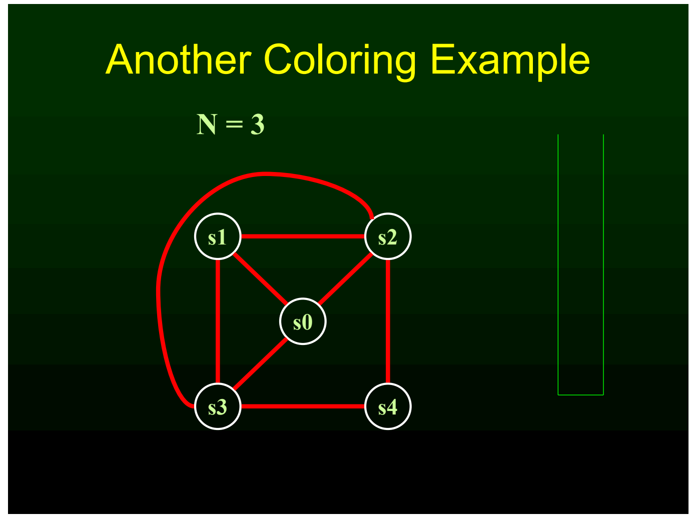
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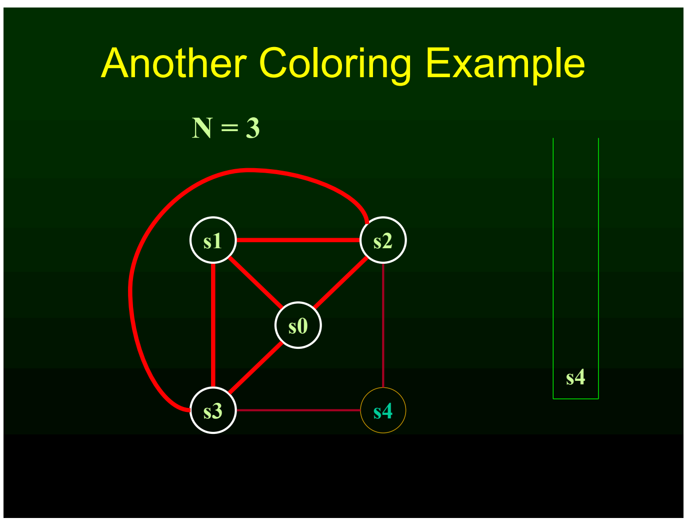
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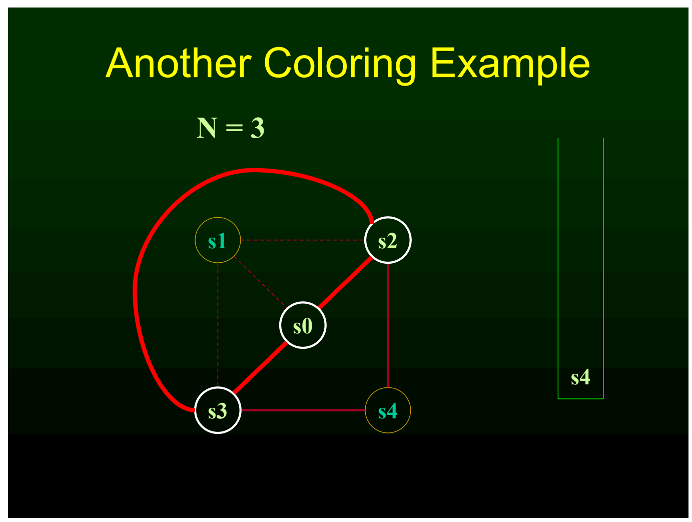
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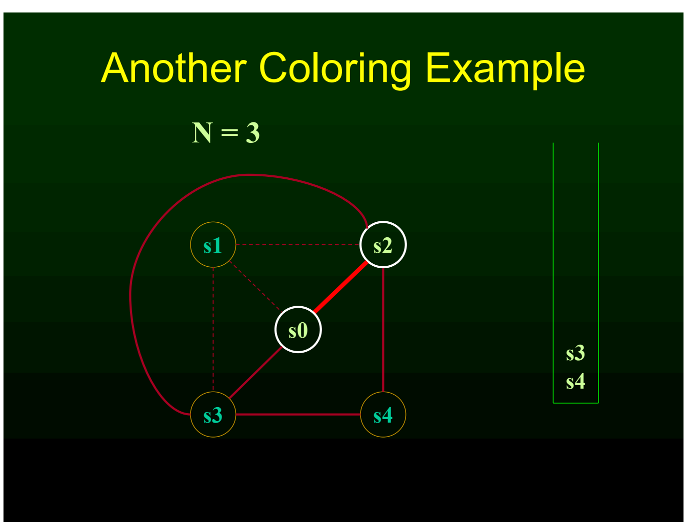
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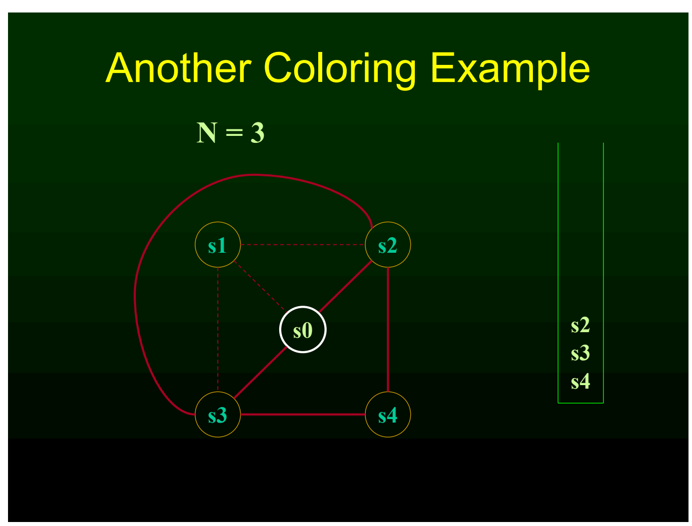
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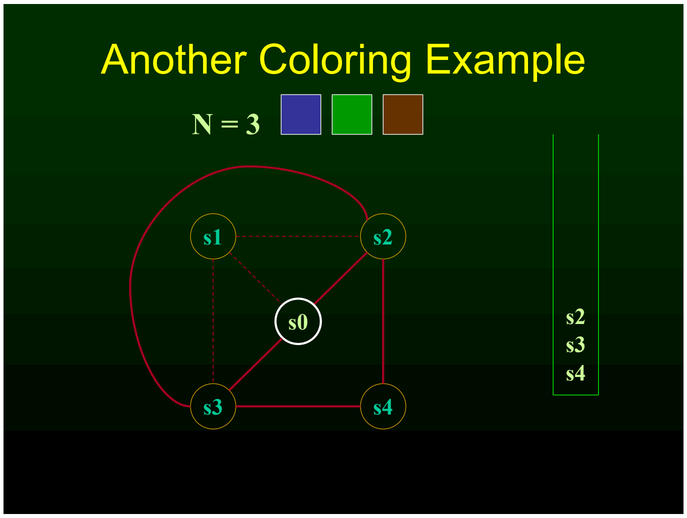
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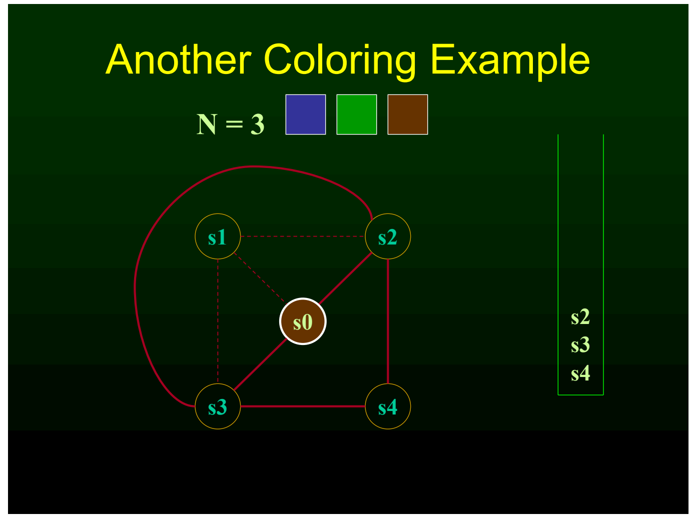
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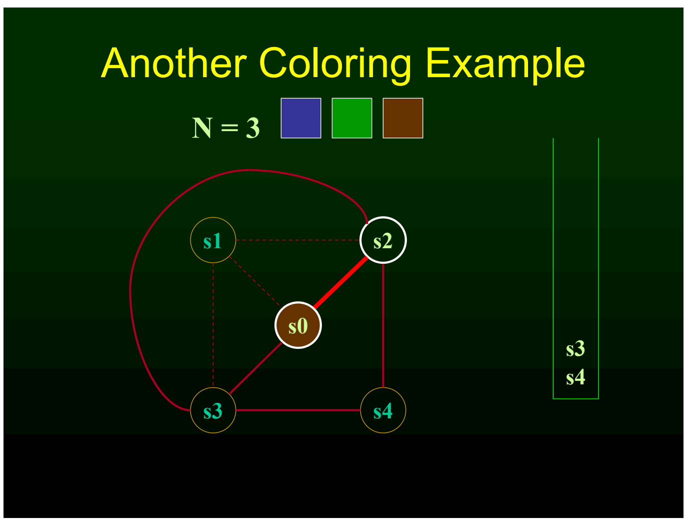
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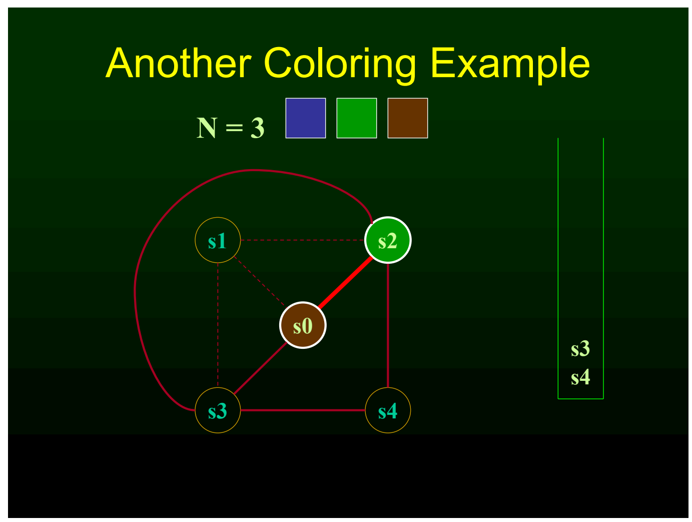
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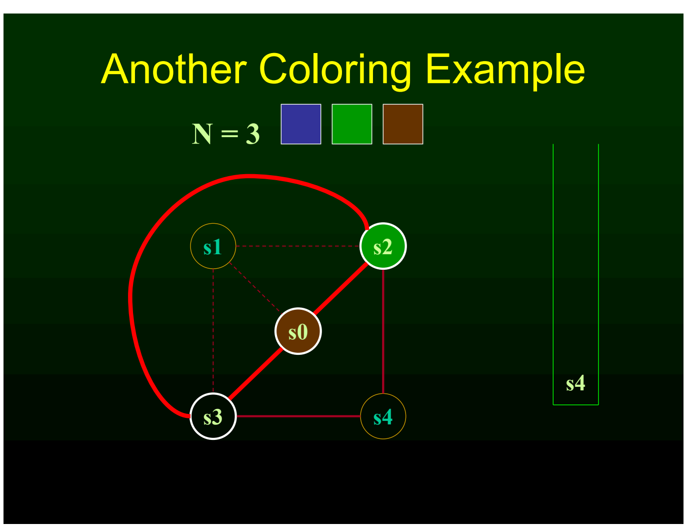
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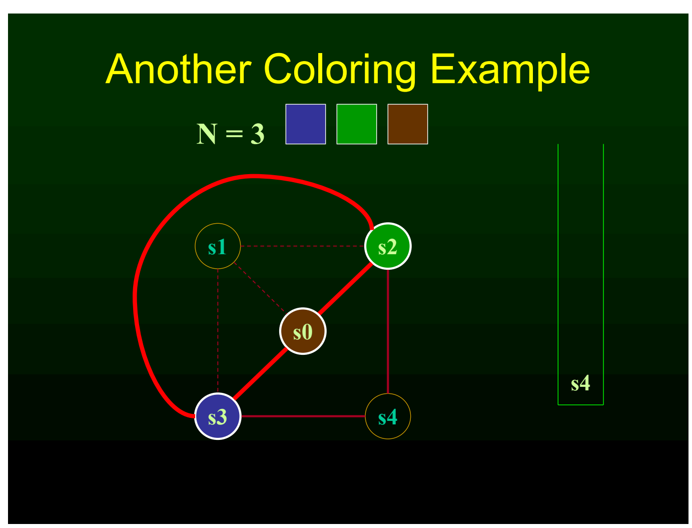
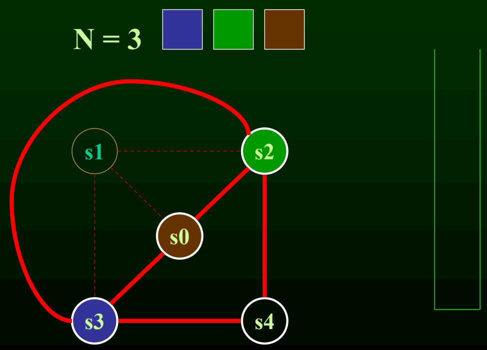
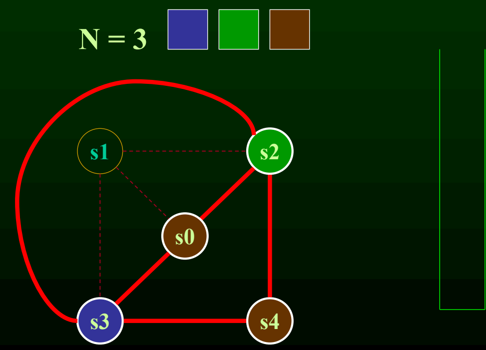
s4
s4


Another Coloring Example
N = 3
s1
s2
s0
s3
s4
s4

Another Coloring Example



Another Coloring Example



What Now?

- Option 1
  - Pick a web and allocate value in memory
  - All defs go to memory, all uses come from memory
- Option 2
  - Split the web into multiple webs
- In either case, will retry the coloring

Which web to pick?

- One with interference degree >= N
- One with minimal spill cost (cost of placing value in memory rather than in register)
- What is spill cost?
  - Cost of extra load and store instructions

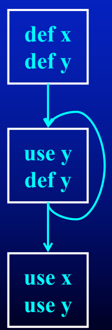
Ideal and Useful Spill Costs

- Ideal spill cost - dynamic cost of extra load and store instructions. Can't expect to compute this.
  - Don't know which way branches resolve
  - Don't know how many times loops execute
  - Actual cost may be different for different executions
- Solution: Use a static approximation
  - profiling can give instruction execution frequencies
  - or use heuristics based on structure of control flow graph

One Way to Compute Spill Cost

- Goal: give priority to values used in loops
- So assume loops execute 10 or 100 times
- Spill cost =
  - sum over all def sites of cost of a store instruction times 10 to the loop nesting depth power, plus
  - sum over all use sites of cost of a load instruction times 10 to the loop nesting depth power
- Choose the web with the lowest spill cost

Spill Cost Example



Spill Cost For x
storeCost+loadCost

Spill Cost For y
9*storeCost+9*loadCost

With 1 Register, Which Variable Gets Spilled?

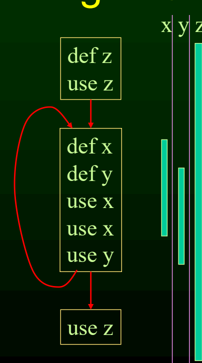
Outline

- Overview of procedure optimizations
- What is register allocation
- A simple register allocator
- Webs
- Interference Graphs
- Graph coloring
- Splitting
- More optimizations

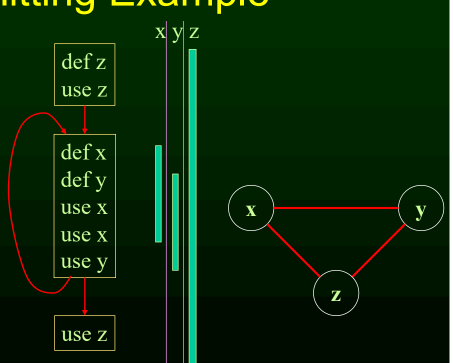
Splitting Rather Than Spilling

- Split the web
  - Split a web into multiple webs so that there will be less interference in the interference graph making it N-colorable
  - Spill the value to memory and load it back at the points where the web is split

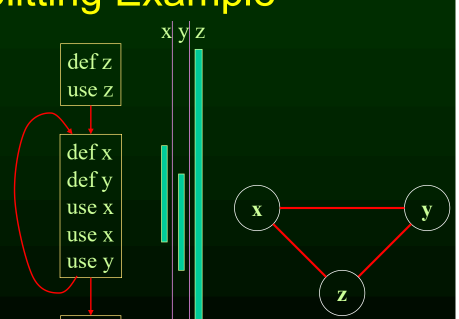
Splitting Example



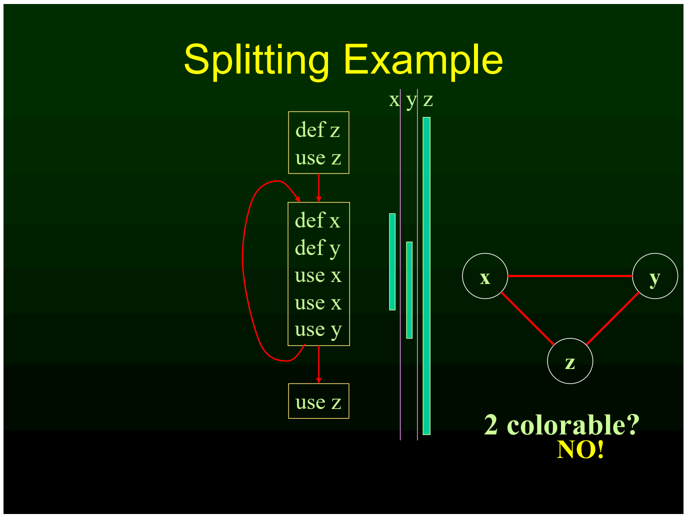
Splitting Example

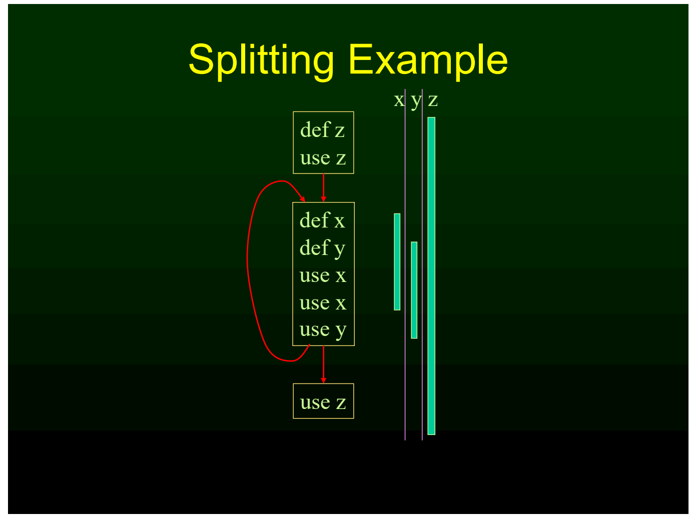


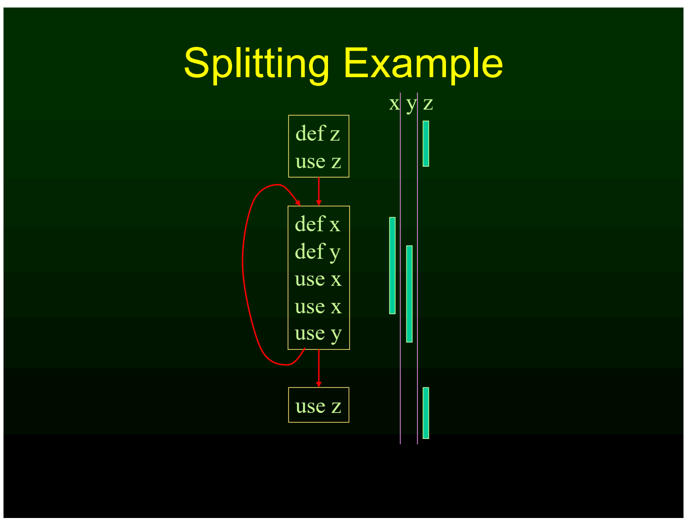
Splitting Example



2 colorable?


Splitting Example
x y z
def z
use z
def x
def y
use x
use x
use y
use z
x
y
z
2 colorable?
NO!


Splitting Example
x y z
def z
use z
def x
def y
use x
use x
use y
use z


Splitting Example
x y z
def z
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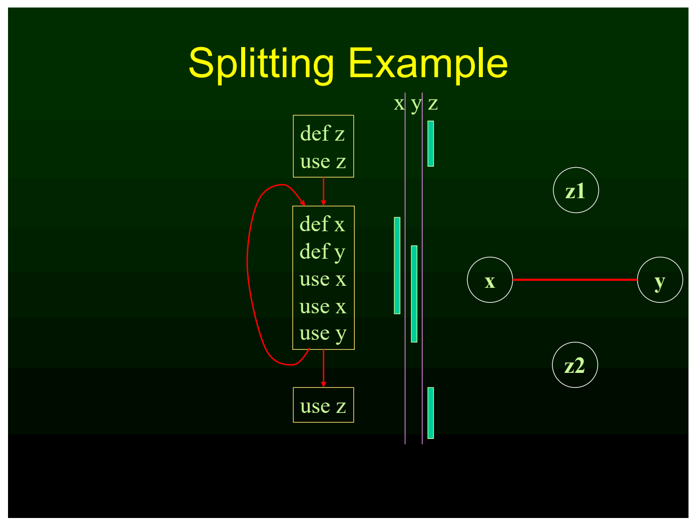
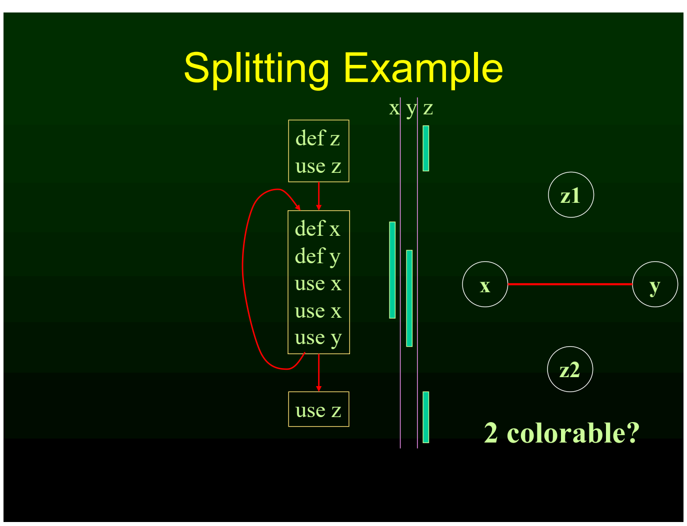
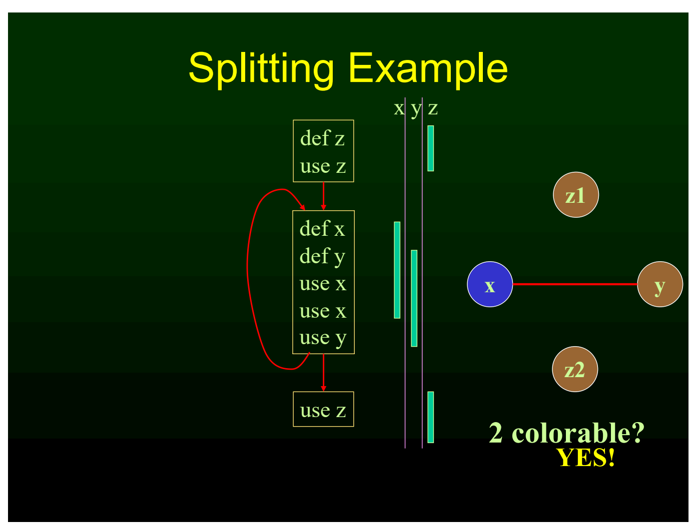
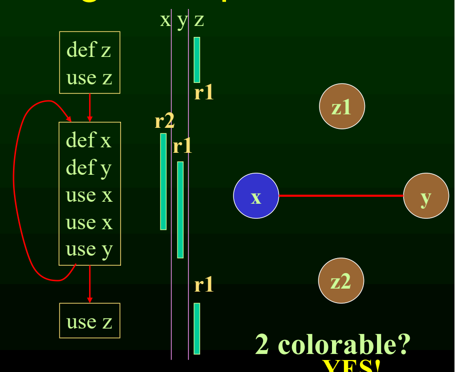

Splitting Example
x y z
def z
use z
def x
def y
use x
use x
use y
use z
z1
x
y
z2


Splitting Example
x y z
def z
use z
def x
def y
use x
use x
use y
use z
z1
x
y
z2
2 colorable?

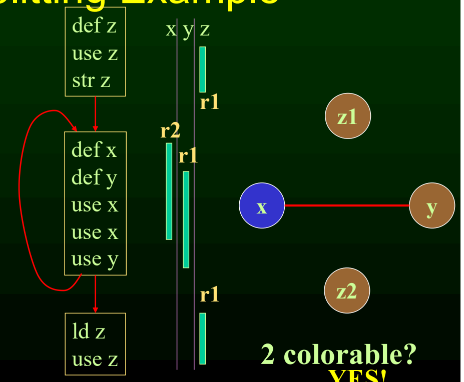

Splitting Example
x y z
def z
use z
def x
def y
use x
use x
use y
use z
z1
x
y
z2
2 colorable?
YES!

Splitting Example



2 colorable?
YES!

Splitting Example



2 colorable?
YES!

Splitting Heuristic

- Identify a program point where the graph is not R-colorable (point where # of webs > N)
  - Pick a web that is not used for the largest enclosing block around that point of the program
  - Split that web at the corresponding edge
  - Redo the interference graph
  - Try to re-color the graph

Cost and benefit of splitting

- Cost of splitting a node
  - Proportional to number of times splitted edge has to be crossed dynamically
  - Estimate by its loop nesting
- Benefit
  - Increase colorability of the nodes the splitted web interferes with
  - Can approximate by its degree in the interference graph
- Greedy heuristic
  - pick the live-range with the highest benefit-to-cost ration to spill

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Further Optimizations

- Register coalescing
- Register targeting (pre-coloring)
- Presplitting of webs
- Interprocedural register allocation

Register Coalescing

- Find register copy instructions sj = si
- If sj and si do not interfere, combine their webs
- Pros
  - similar to copy propagation
  - reduce the number of instructions
- Cons
  - may increase the degree of the combined node
  - a colorable graph may become non-colorable

Register Targeting (pre-coloring)

- Some variables need to be in special registers at a given time
  - fist 6 arguments to a function
  - return value
- Pre-color those webs and bind them to the right register
- Will eliminate unnecessary copy instructions

Pre-splitting of the webs

- Some live ranges have very large "dead" regions.
  - Large region where the variable is unused
- Break up the live ranges
  - need to pay a small cost in spilling
  - but the graph will be very easy to color
- Can find strategic locations to break-up
  - at a call site (need to spill anyway)
  - around a large loop nest (reserve registers for values used in the loop)

Interprocedural register allocation

- saving registers across procedure boundaries is expensive
  - especially for programs with many small functions
- Calling convention is too general and inefficient
- Customize calling convention per function by doing interprocedural register allocation

Summary

- Register Allocation
  - Store values in registers between def and use
  - Can improve performance substantially
- Key concepts
  - Webs
  - Interference graphs
  - Colorability
  - Splitting