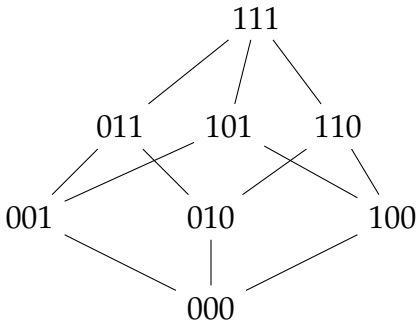
1 Lattices

Let $L = \{000, 001, 010, 011, 100, 101, 110, 111\}$, with $a \leq b$ defined by $a \,\&\, b = a$.

1.1 Hasse Diagram

Draw the Hasse Diagram for this lattice. Recall that a lattice element y covering an element x is represented by an edge from y to x.

Answer:



1.2 Is this lattice complete?

Answer: Yes

1.3 What is the top element?

Answer: 111

1.4 What is the bottom element?

Answer: 000

1.5 Evaluate the following:

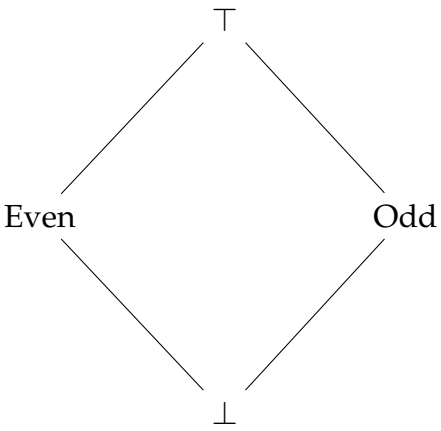
Expression	Value
$100 \wedge 110$	
$011 \vee (001 \wedge 111)$	
$010 \vee 001$	
$001 \wedge (100 \vee 010)$	

Answer:

Expression	Value
$100 \wedge 110$	100
$011 \vee (001 \wedge 111)$	011
$010 \vee 001$	011
$001 \wedge (100 \vee 010)$	000

2 Parity Analysis

We track the parity of variables using the lattice:



For $a = b + c$, where a, b, and c are members of our new lattice, we define the transfer function:

$$f([a \to p_1,\ b \to p_2,\ c \to p_3]) = [a \to p_2 \oplus p_3,\ b \to p_2,\ c \to p_3]$$

Where p_1, p_2, and p_3 are elements of the base lattice.

2.1 Fill in the table for the $\oplus$ operator

$\oplus$	$\bot$	Even	Odd	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
Even	$\bot$			$\top$
Odd	$\bot$			$\top$
$\top$	$\bot$	$\top$	$\top$	$\top$

Answer:

$\oplus$	$\bot$	Even	Odd	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
Even	$\bot$	Even	Odd	$\top$
Odd	$\bot$	Odd	Even	$\top$
$\top$	$\bot$	$\top$	$\top$	$\top$

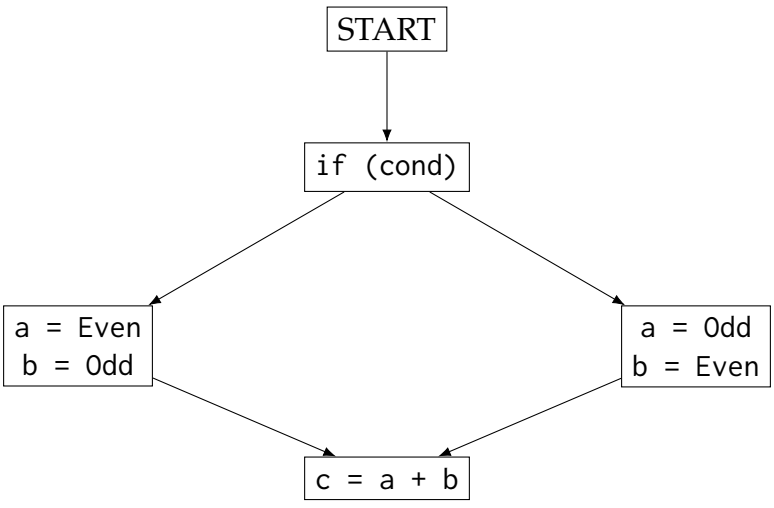
2.2 Finish the transfer function of a statement `c = a`

$$f([a \to p_1, b \to p_2, c \to p_3]) = [a \to \_\_\_\_\_, \quad b \to \_\_\_\_\_, \quad c \to \_\_\_\_\_]$$

Answer:

$$= [a \to p_1, \quad b \to p_2, \quad c \to p_1]$$

Suppose we are performing parity analysis on the following control flow graph:



2.3 What is the lattice point associated with the program point after the node `c = a + b`?

$$[a \to \_\_\_\_\_, \quad b \to \_\_\_\_\_, \quad c \to \_\_\_\_\_]$$

Answer:

$$= [a \to \top, \quad b \to \top, \quad c \to \top]$$

2.4 As a human, what is the most precise parity information you can determine for the program point after the node `c = a + b`?

$$[a \to \_\_\_\_\_, \quad b \to \_\_\_\_\_, \quad c \to \_\_\_\_\_]$$

Answer:

$$= [a \to \top, \quad b \to \top, \quad c \to \text{Odd}]$$